Vectors & Transformations Practice Test

1. Eigenvalues, Eigenvectors, & Invariant Lines

Consider the transformation given by the matrix:

\[
\begin{bmatrix}
3 & 1 \\
1 & 3 \\
\end{bmatrix}
\]

(a) Draw the image of the unit square under this transformation.

(b) Find the eigenvalues, eigenvectors, and invariant lines of this transformation matrix.

(c) Use your results from part (b) to describe the transformation.

(d) What are the coordinates of (1, 0) and (0, 1) in the Eigen axis system.
2. Lines & Planes in 3-Space

(a) Find the equation of the line through \( P(1,-1,2) \) which is perpendicular to the plane \( 3x + 2y - z = 6 \). Recall that this needs to be a parametric equation.

(b) Find the coordinates of the point, \( S \), where this line meets the plane.

(c) Find the distance between the two points \( P \) and \( S \).
(d) Repeat the above process for the point \((x_1, y_1, z_1)\) and the plane \(ax + by + cz = d\).

(e) The distance between two skew lines is the length of their mutual perpendicular. Use a method similar to the above to find the distance between the lines below. Sketch a diagram to show what you are finding and to give you added direction in your work.

\[
\begin{align*}
\begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\
\begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}
\end{align*}
\]
3. More Lines & Planes in 3-Space

(a) Find the intersection of the following three planes. If it’s a plane, show the equation of the plane. If it’s a line, find the equation of the line. If it’s a point, find the coordinates of the point. If they’re parallel, state that and show why.

\[
\begin{align*}
2x - 2y + 6z &= 12 \\
2x + y - 2z &= 8 \\
-x + y - 3z &= -6
\end{align*}
\]

(b) Find the intersection of the following two lines. If they’re parallel, show why. If they’re perpendicular, show why. If they’re skew, show why.

\[
\begin{align*}
\begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \\
\begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} + s \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}
\end{align*}
\]
4. Transformation Matrices

For each of the following transformation matrices, list (in order) what transformation(s) is/are happening. Show your work. You may use the attached grids to aid you in the process.

(a) \[
\begin{bmatrix}
-1 & 3 \\
3 & -9 \\
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
7 & -24 \\
-24 & -7 \\
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
2 & 8 & -3 \\
0 & 2 & 5 \\
0 & 0 & 1 \\
\end{bmatrix}
\]
5. Vectors & Matrices

(a) Given the line $x + y = 2$ find the vector equation of a line perpendicular to this line through the point (-1, 2).

(b) This new line (from part (a)) is then reflected over the line $y = \frac{1}{2}x$. Write the transformation matrix for this reflection.

(c) Find the equation of the image line from part (b).
6. Transformation Matrices

For each of the following matrices, using row reduction, determine which simple transformations (no more than four), in order, achieve the same effect as the original transformation. The attached grids are here for you to check your work.

(a) \[
\begin{bmatrix}
5 & -8 \\
1 & -2
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
4 & 2 \\
4 & 3
\end{bmatrix}
\]