Using Matrices to Represent Data

A matrix is an array of numbers where the position of the number in the array is important. Matrices are very important in higher levels of mathematics, and the next few sections will give you an introduction to some of their properties and applications. In this section we will study matrices as a tool to efficiently organize and manipulate data. In later sections we will see that matrices provide another way to represent and solve systems of linear equations.

Example
The towns of Mitchell, Bedford, Shoals, and Oolitic all participate in a county-wide softball league. An accountant for the league uses a spreadsheet to keep track of each town’s equipment orders. Parts of that spreadsheet are shown below.

<table>
<thead>
<tr>
<th>Mitchell Softball Teams</th>
<th>Shoals Softball Teams</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Uniforms</strong></td>
<td><strong>Uniforms</strong></td>
</tr>
<tr>
<td><strong>Bats</strong></td>
<td><strong>Bats</strong></td>
</tr>
<tr>
<td><strong>Balls</strong></td>
<td><strong>Balls</strong></td>
</tr>
<tr>
<td>Men’s</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>120</td>
</tr>
<tr>
<td>Women’s</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bedford Softball Teams</th>
<th>Oolitic Softball Teams</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Uniforms</strong></td>
<td><strong>Uniforms</strong></td>
</tr>
<tr>
<td><strong>Bats</strong></td>
<td><strong>Bats</strong></td>
</tr>
<tr>
<td><strong>Balls</strong></td>
<td><strong>Balls</strong></td>
</tr>
<tr>
<td>Men’s</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>300</td>
</tr>
<tr>
<td>Women’s</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>210</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Store A</th>
<th>Store B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniforms</td>
<td>$59.00</td>
<td>$50.00</td>
</tr>
<tr>
<td>Bats</td>
<td>$31.00</td>
<td>$34.99</td>
</tr>
<tr>
<td>Balls</td>
<td>$1.50</td>
<td>$2.25</td>
</tr>
</tbody>
</table>

To represent the Oolitic data as a matrix, you drop the labels and put brackets around the numbers. The number “9” on the bottom row of this matrix means “Oolitic’s softball team needs 9 women’s bats.”

\[
\begin{bmatrix}
11 & 6 & 80 \\
15 & 9 & 100
\end{bmatrix}
\]

The exercises in this section will help you discover some properties of matrices in the context of this softball league example.
Exercises
Use the information in the tables to complete the exercises.

<table>
<thead>
<tr>
<th>Mitchell Softball Teams</th>
<th>Shoals Softball Teams</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniforms</td>
<td>Bats</td>
</tr>
<tr>
<td>Men's</td>
<td>24</td>
</tr>
<tr>
<td>Women's</td>
<td>23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bedford Softball Teams</th>
<th>Oolitic Softball Teams</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>Men's</td>
<td>31</td>
</tr>
<tr>
<td>Women's</td>
<td>33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Store A</th>
<th>Store B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>Price</td>
</tr>
<tr>
<td>Uniforms</td>
<td>$59.00</td>
</tr>
<tr>
<td>Bats</td>
<td>$31.00</td>
</tr>
<tr>
<td>Balls</td>
<td>$1.50</td>
</tr>
</tbody>
</table>

1. Often we may want to manipulate the information contained in a matrix. To this end, it is sometimes useful to **add** matrices. Just add the corresponding numbers like this:

\[
\begin{bmatrix}
11 & 6 & 80 \\
15 & 9 & 100
\end{bmatrix} + \begin{bmatrix}
31 & 19 & 300 \\
33 & 15 & 210
\end{bmatrix} = \begin{bmatrix}
11 + 31 & 6 + 19 & 80 + 300 \\
15 + 33 & 9 + 15 & 100 + 210
\end{bmatrix} = \begin{bmatrix}
42 & 25 & 380 \\
48 & 24 & 310
\end{bmatrix}
\]

   a. What does the number “24” mean in the context of this problem?

   b. Re-write the final matrix below and add labels to clearly show what the numbers represent.
2. Evaluate the following:

\[
\begin{bmatrix}
11 & 6 & 80 \\
15 & 9 & 100
\end{bmatrix} +
\begin{bmatrix}
31 & 19 & 300 \\
33 & 15 & 210
\end{bmatrix} +
\begin{bmatrix}
24 & 13 & 120 \\
23 & 11 & 100
\end{bmatrix} +
\begin{bmatrix}
24 & 13 & 120 \\
28 & 12 & 190
\end{bmatrix} =
\begin{bmatrix}
\_ & \_ & \_ \\
\_ & \_ & \_
\end{bmatrix}
\]

What does the final matrix represent? Why might the person in charge of the county’s equipment budget be interested in this matrix?

3. What could the following matrix represent:

\[
\frac{1}{2}
\begin{bmatrix}
24 & 13 & 120 \\
28 & 12 & 190
\end{bmatrix} =
\begin{bmatrix}
12 & 6.5 & 60 \\
14 & 6 & 95
\end{bmatrix}
\]

4. Suppose Bedford wanted to triple their entire equipment order. Write a matrix to represent this order.

5. The accountant for the league can place orders at either Store A or Store B (let’s say they have to buy everything from the same store). How much will it cost to buy the men’s equipment for the town of Oolitic from each store?

6. Matrix Multiplication can be used for the type of computation you performed in Exercise 5. You “combine” a row from the first matrix with a column from the
second matrix to get one entry in the “answer” matrix. It looks a little weird at first, but you should see that it does exactly the same computations you did in the previous problem:

\[
\begin{bmatrix}
11 & 6 & 80 \\
15 & 9 & 100
\end{bmatrix}
\begin{bmatrix}
59.00 & 50.00 \\
31.00 & 34.99 \\
1.50 & 2.25
\end{bmatrix}
= 
\begin{bmatrix}
11 \cdot 59.00 + 6 \cdot 31.00 + 80 \cdot 1.50 \\
15 \cdot 59.00 + 9 \cdot 31.00 + 100 \cdot 1.50
\end{bmatrix}
= 
\begin{bmatrix}
995.00 & 939.94 \\
1314.00 & 1289.91
\end{bmatrix}
\]

a. What does the number 1314.00 in the resulting matrix represent?

b. Complete the remaining labels at right to show your understanding.

7. Perform the indicated matrix multiplication and then describe what the entries in the answer matrix mean in the context of the softball league data.

\[
\begin{bmatrix}
24 & 13 & 120 \\
28 & 12 & 190
\end{bmatrix}
\begin{bmatrix}
59.00 & 50.00 \\
31.00 & 34.99 \\
1.50 & 2.25
\end{bmatrix}
\]