Assignment #4: Solving Equations

Here's the difference between arithmetic and algebra. In arithmetic, you take known quantities and you manipulate them. Example: if you bought 40 peaches for $3.60, how many more could you get with the rest of your $5 bill? Answer: 40 into 360 is 9, so the peaches cost 9¢ apiece. 500 subtract 360 is 140, so we divide 140 by 9 to get 15 and 5/9 peaches. Note how in this instance we are manipulating known quantities.

In **algebra**, you take a leap of abstraction. You manipulate unknown quantities, symbolized by variables, until they become known. Example: how much past 6 o'clock will the big hand of the clock be exactly above the little hand? You cannot answer this readily by manipulating known quantities. So another approach would be to symbolize the unknown, i.e. how far the big hand goes, by x. Then the little hand must have gone 1/12 x, because the little hand goes 1/12 as fast as the big hand. Then say: "To catch the little hand, the big hand must go 30 plus whatever the little hand goes." Thus $x = 30 + 1/12\ x$, and we have an equation which we can solve to find x, as we will practice below (and throughout this course). This clever idea of manipulating unknown quantities, symbolized by variables, until they become known, is a distinguishing feature of algebra (and it's incredibly fun).

Vocabulary:

An expression frequently is composed of parts that are to be added or subtracted. These parts are called **terms**. In the expression $2x + xy^3 - 7$, we have three terms. The multiplier of a term is called its **coefficient**. The coefficient of the first term in the expression above is 2; the coefficient of the second term is 1. The letters which stand for numbers are **variables**, and the term without any variable is called a **constant** term. The **exponent** of y in the second term is 3.

When two expressions are declared to be equal, using the = sign, the declaration is called an **equation**. A **linear equation** is one where the highest exponent in any term is 1. (We'll see why this type of equation is called linear in the chapter on graphing.) Example: $2x + 4 = 5x - 5$. **Solving** an equation means finding the value(s) of the variable which make the equation true. In the example above, $x=3$ makes the equation true; 3 is called a **solution** to the equation. Some equations have more than one solution. The set of all solutions is called, not surprisingly, the **solution set**. If there are no solutions, the solution set is the **empty set**, symbolized by {} or by ø. If <u>any</u> real number is a solution, the solution set is symbolized by $\Re$. For example, the equation $x=x$ has $\Re$ for a solution set.

Note the difference between equations and expressions. You <u>solve</u> an equation; you <u>evaluate or simplify</u> an expression.

Example:

$$\begin{array}{rcl} 7x+12 & = & -2x+4 \\ +2x & & +2x \end{array}$$

Add 2x to get the terms with x together.

$$9x+12 = 4$$
$$\begin{array}{rr} -12 & -12 \end{array}$$

Subtract 12 to get the term with x alone on one side of the equation.

$$9x = -8$$

$$\frac{9x}{9} = \frac{-8}{9}$$

Divide by the coefficient 9.

$$x = \frac{-8}{9}$$

<u>Allowable transformations of equations</u>

The following tools for solving equations are illustrated below:

Addition to (or subtraction from) both sides of the equation.
Division (or multiplication) by a non-zero number on both sides of the equation.
Distribution, and its converse, combining **like terms** (like $7x$ and $-13x$).

The general technique for solving equations will be to:

1) simplify each side of the equation (i.e. distribute and combine like terms),
2) use addition (or subtraction) to collect the x terms on one side,
3) use division (or multiplication) to find x.

Example:

$2(3x+7) - (2-x) = -2(x-3) +1$	Original problem
$2(3x+7)+-(2+-x) = -2(x+-3)+1$	Change - to +-
$2(3x+7)+-1(2+-x) = -2(x+-3)+1$	Change -(2+-x) to -1(2+-x)
$6x+14+-2+x = -2x+6+1$	Distribute
$7x+12 \quad = \quad -2x+7$	Combine like terms
$+2x \qquad\qquad +2x$	Collect x's on one side...
$9x+12 = 7$	
$\quad -12 \quad -12$	...and the constants on the other side
$9x = -5$	Divide by the coefficient
$\frac{9x}{9} = \frac{-5}{9}$	
$x = \frac{-5}{9}$	And finally we have x.

<u>A few fine points</u>:

- We observed earlier that the opposite (i.e. the - sign) of a number is the same as -1 · the number. Thus -(-2) is -1·-2 which is 2, and -x is the same as -1x. So -(2x-3) can be seen as -1·(2x-3) and distributed to get -2x+3.

- Also -3· -x can be seen as -3· –1·x which is 3·x or 3x.

- Furthermore recall that x is 1x. So 3x+x is 3x+1x or (3+1)x, which is 4x.

- Earlier we said that a negative times a negative is a positive. The distributive axiom allows us to establish this a bit more formally:

$-1 + 1 = 0$	Inverse axiom
$-1(-1 + 1) = 0$	Multiply both sides by -1
$(-1)(-1) + (-1)(1) = 0$	Distributive Axiom
$(-1)(-1) + -1 = 0$	Identity Axiom for multiplication
$(-1)(-1) = 1$	Add 1 to both sides

Problems

Solve

1. a. $2(c+31)=17c+2$ b. $9(x-8)=2(x+3)-1$ c. $7(y+4)=3(9y+12)-19y$
2. a. $2-4(y+5)=y-1+8$ b. $6-c+23=5(c-4)+c$ c. $2x+7(x+7)-2=1-8x-5$
3. In common English, what do the inverse axioms say?
4. In common English, what do the identity axioms say?
5. The larger number is 2 more than 4 times the smaller one. The smaller one is 8 less than the larger. Find both numbers.
6. How many minutes past 6 o'clock will the hands of a clock be exactly together?

(L): Write 9 in binary, 10 in binary, 3 in trinary, 8 in octal, and 7 in octal.