Assignment #3: The Field Axioms

An **axiom** is a rule or property that we accept without proof as a starting point when considering some topic. The real numbers obey the following eleven axioms; therefore when we want to prove some proposition about the real numbers, we will often use these axioms. Whenever a set of numbers obeys these eleven axioms, the set is called a **field**; thus these axioms are called the **field axioms**, and the real numbers are a field.

Here is a concise way to write the eleven field axioms. In the following list, x, y, and z are any real numbers.

Closure axioms	$x+y$ is real;	$x \cdot y$ is real.
Commutative axioms	$x+y = y+x$;	$x \cdot y = y \cdot x$
Associative Axioms	$(x+y)+z=x+(y+z)$;	$(x \cdot y) \cdot z=x \cdot (y \cdot z)$
Distributive Axiom	$x(y+z)=xy+xz$	
Identity Axioms	$x+0=x$	$x \cdot 1=x$
Inverse Axioms	$x+-x=0$	$x \cdot \frac{1}{x} = 1$ (if $x \neq 0$)

Translation into English:

Closure axioms	If you add two real numbers, you get another real number. If you multiply two real numbers, you get another real number.
Commutative axioms	You can add in either direction; the result is the same. You can multiply in either direction; the result is the same.
Associative Axioms	When adding several numbers, you may group as you please. When multiplying several numbers, you may group as you please.

Distributive Axiom	You may add a group, then multiply; or multiply to each member of the group, then add. The result is the same.
Identity Axioms	There exists an additive identity (0) and a multiplicative identity (1). This means that there is a number called 0 which can be added to any x to get x itself. (Hence the word *identity*.) Similarly, there is a number called 1 which can be multiplied by any x to get x itself.
Inverse Axioms	The inverse axiom says that for every real number x, there is an additive inverse (-x), and if $x \neq 0$, there is a multiplicative inverse (1/x). In other words, for every x there is a -x which we can add to x to get the identity element, 0. Also for every non-zero x, there is a reciprocal 1/x which can be multiplied by x to get the identity element, 1.

Problems

1. Classify the following numbers into as many categories as you can. (Use the following list: imaginary, real, rational, irrational, integer, non-integer, radical, transcendental.)

   4. $\overline{234}$ -3 -3.2 $\sqrt{7}$ $\sqrt{.49}$ $\sqrt{-4}$ π

2. In common English, what do the closure axioms mean?

3. In common English, what do the commutative axioms mean?

4. Find a number that is 14 less than twice the sum of itself and 3. Classify this number as fully as possible. (Use the following list: imaginary, real, rational, irrational, integer, non-integer, radical, transcendental.)

5. Prove that each of the following is rational by finding an equivalent ratio of integers:

   5. $\overline{23}$ b) $1.34\overline{7}$ c) 12.45 d) 0 e) $4.33\overline{12345}$

6. On the radio show *Car Talk*, Click and Clack claim that if you take apart and reassemble a Volkswagen carburetor enough times, you will eventually have enough spare parts left over to make two carburetors! What field axiom does this violate?

(L): Find the next three items in this sequence: J^{K}, L^{J}, R^{N}, G^{F}, __, __, __