Assignment #22: Solving In General (Cramer's Rule). Inconsistent and Dependent Equations

Example: Solve 5x+8y+6=4x+6y+10 and 4x+7y+5=-2x+-5y+11

Solution: It helps to have moved the variables all to one side and the constants to the other side before transforming the equations in preparation for adding. When we're done moving things around, the system looks like this:

$$x+2y=4 \quad \text{and} \quad 6x+12y=6$$

Let's multiply the first equation by -6:

$$-6x+-12y=-24 \quad \text{and} \quad 6x+12y=6$$

Now we add the two equations:

$$0 = -18$$

Hey! This will <u>never</u> be true, so the system is **inconsistent**. (If we had gotten something that is always true, like -18 = -18, we would have said that this is <u>always</u> true, so the system would be **dependent**.)

Cramer's Rule

(after Gabriel Cramer, b. July 31, 1704, Geneva, Switzerland; d. January 4, 1752, Bagnol-sur-Ceze, France)

This method involves using a general solution to:

$$ax + by = c$$
$$dx + ey = f$$

By using the combinations approach, a general solution can be found, as shown in Appendix J:

Cramer's Rule: the solutions to

$$ax + by = c$$
$$dx + ey = f$$

are

$$x = \frac{ce-bf}{ae-bd}$$

$$y = \frac{af-cd}{ae-bd}$$

A result like this can only be contemplated with wonder and delight! It is remarkable that we can make such a sweeping and yet specific statement regarding the answers to <u>any</u> system of two equations and two unknowns.

Note that if ae-bd=0 and ce does not equal bf, we have no answer (**inconsistent equations**). This reflects the fact that ae=bd, i.e. that a/b = d/e, i.e the equations are multiples of one another in their coefficients. Divide the first equation by b and the second by e, and we can see that the system can have <u>no</u> <u>answer</u>.

If ae=bd <u>and</u> ce=bf, then a/d = b/e = c/f. In other words, the entire second equation is a multiple of the first, so the two equations are really the same, and hence there are many solutions (**dependent equations**).

The solutions

$$x = \frac{ce-bf}{ae-bd} \qquad y = \frac{af-cd}{ae-bd}$$

can be expressed simply as the ratio of **determinants** of the appropriate **matrices** and this will be discussed in class. For example, if one is solving for x, one replaces the x coefficients by the constants to form the numerator matrix

$$\begin{bmatrix} c & b \\ f & e \end{bmatrix}$$

which one divides by the determinant of the original coefficients matrix

$$\begin{bmatrix} a & b \\ d & e \end{bmatrix}$$

In class, we will generalize this to Cramer's rule for three-variable systems, again using matrices and determinants.

<u>Problems</u>

Solve for both variables by substitution.

1. a. 7x+-3y=-27 x+y=-1 b. x+-5y=-62 6x+-5y=-72

Solve for both variables by using Cramer's Rule. (See above.) Be alert for **inconsistent** or **dependent** equations.

2. a. -x+-3y=9 2x+5y=-15 b. -x+3y=-11 2x+-5y=19

Solve for both variables. Be alert for **inconsistent** or **dependent** equations. Note: it helps to have moved the variables all to one side and the constants to the other side before transforming the equations in preparation for adding them.

3. x+2y+4=3x+5y-9
   4x+2y+6=7x+9y-21
4. 3x+6y+7=4x+9y+1
   4x+8y+3=7x+8y-6
5. 5x+8y+6=4x+6y+10
   4x+7y+5=-2x+-5y+11

(L): Burger King is going to have a new offering called "Burguettes", which are small burgers which can be ordered only in "paks". A small pak has 7 Burguettes and a large pak has 15 burguettes. What is the <u>largest</u> number of Burguettes that <u>cannot</u> be ordered by making some combination of small and large paks?