

Assignment #20: Solving by Linear Combinations

Example: Solve $6x+7y=114$ $3x+12y=108$

Recall that two equations can be added to result in another valid equation. Here we transform the equations so that when we add them, one of the variables is eliminated. (The technique is often referred to as solving by "taking linear **combinations**" of the equations. Authors who believe in plain speech occasionally call this method "adding the equations".)

$$\begin{array}{r} 6x + 7y = 114 \text{ -----copy-----} \rightarrow 6x + 7y = 114 \\ 3x + 12y = 108 \text{ -- multiply by -2 ----} \rightarrow -6x + -24y = -216 \\ \hline -17y = -102 \\ y = 6 \end{array}$$

We then substitute $y=6$ back into any of the two-variable equations to get $x=12$. To illustrate, we will put $y=6$ into the equation $6x+7y=114$.

$$\begin{aligned} 6x+42 &= 114 \\ 6x &= 72 \\ x &= 12 \end{aligned}$$

So placebos mentioned above cost \$12 and the gazebos cost \$6.

Reminders: It helps to have moved the variables all to one side before transforming the equations in preparation for adding them. Remember that if we transform one side of an equation by multiplication, we must also multiply on the other side.

This technique will work for three-variable systems too. When we try it, we will aim to eliminate a variable with two of the equations, then eliminate the same variable with two other equations. Then we will have a nice, familiar two-variable system.

Problems

Find the solution set and graph on a number line.

1. a. $|2x+1| \leq 3$

b. $|2x+5| < 7$

c. $|9x+9| > 7$

2. a. $|-5x-2| \leq 0$

b. $|-3x-6| \geq 3$

c. $|-3x-1| > -4$

Solve for both variables by adding the equations.

3. a. $-5x+5y=5$ $5x+3y=19$

b. $8x+3y=-9$ $-8x-6y=-6$

Solve for both variables by linear combinations.

4. a. $3x+5y = -9$ $6x+12y=48$

b. $2x+3y = -4$ $6x+4y=14$

5. a. $4x+5y=27$ $2x+12y=-30$

b. $3x+4y = -11$ $-2x+2y=7$

6. $|x-7|$ is sometimes read by Tess as "the distance from x to 7". Are they the same thing? If so, how would Tess read $|x+3|$?

(L): $\sqrt{\sqrt{\sqrt{\sqrt{x}}}} = 2$. Find x .