Assignment #2: The Real Numbers

What is a "real number"? Since numbers are ideas, it's hard to call any of them "real" in the usual sense, but the ones that are the most familiar to us are the ones we call real. These are the numbers we normally deal with, and since they are used for counting and measuring real things, they are called the **real** numbers. As a definition, we say that the real numbers are all the numbers that have a place on the number line. Examples of <u>real</u> numbers are 2, -1.2, π, $\sqrt{2}$, $\frac{2}{7}$

Numbers can be thought of that are not in the set of real numbers, i.e. not on the number line. Some are called the **imaginary** numbers; these involve the square roots of negative numbers.

The real numbers can be divided into the **rational** numbers and the **irrational** numbers. Rational numbers are those that can be expressed as a ratio of two whole numbers (positive or negative). In other words, a rational number can be expressed as a fraction, with whole numbers in the numerator and denominator. Irrational numbers cannot be expressed in this way. Examples of rational numbers are 2, -1.2, $\frac{2}{7}$ and examples of irrational numbers are $\sqrt{3}$, $-\sqrt[3]{5}$, π.

Question: "What about $1.\overline{7}$, with only the 7 repeating? Is that rational?"

Answer: Yes. $1.\overline{7}$ is the same as $\frac{16}{9}$. Try the division $16 \div 9$ to check this.

Question: "What about decimals that go on forever, but don't repeat? Are they rational?"
Answer: No. (How might you prove this?)

The rational numbers can be divided into the **integers** and the **non-integers**. The integers are the whole numbers, and their opposites. Note that zero is a whole number. Some examples of integers are 2, -3, and 0. The integers can be further described as **even**, or **odd**, or **positive** (greater than zero) or **negative** (less than zero). The non-integers are the fractions between the integers.

The irrational numbers can be divided into the **radicals** (which can be expressed with square roots of integers, and other roots of integers) and the **transcendental** numbers (which cannot be expressed only with roots of integers). Examples of radicals are $\sqrt{3}$, $-\sqrt[3]{5}$. Examples of transcendental numbers: π, cos 2° (This book covers numbers like cos 2° in chapter 9.)

Problems

1. What is the difference between an integer and a non-integer?
2. What is the difference between a real number and an imaginary number?
3. What is the difference between a rational number and an irrational number?
4. Show by example that if Nikhil took the square root of an integer, he could get any of the following kinds of numbers: imaginary, rational, irrational, integer.
5. Find a number which is equal to 5 more than 13 times the number itself. Classify this unfortunate number into as many of the categories as you can. (Use the following list: imaginary, real, rational, irrational, integer, non-integer, radical, transcendental.)
6. Prove that $5.\overline{9}$ is rational. (See the example below.)

Example: Prove that $1.\overline{7}$ is rational.

Let $x = 1.\overline{7}$. Then $10x = 17.\overline{7}$. So then 9x, which is 10x - x, is $17.\overline{7} - 1.\overline{7}$, which is 16. So 9x = 16, and thus $x = \frac{16}{9}$. So $1.\overline{7}$ is $\frac{16}{9}$, which is rational because it's the ratio of two integers.

7. The thickness of a piece of notebook paper is about 10^{-4} meters. How many pieces of such paper would it take to make a stack 1 inch high? (If you not only calculate the number of pieces, but also stack up that many pieces, and it actually measures one inch, we'll throw an <u>incredibly</u> short party.)

(L): Adam is at the bottom of a 9.5 foot deep well. Each day he crawls up 3 feet, but each night he slips back 2 feet. If he starts up in the morning of October 1st, on what day will he escape?

(R): Show how, using only a pencil and a straight edge, you can draw one straight line that will cut both rectangles into two equal areas. *(Hint: you may draw other guide lines to help* <u>*locate*</u> *the one line that will cut both rectangles into two equal areas.)*