Assignment #4: Order of Operations

There are some agreements we have to make when reading expressions and doing calculations; otherwise we will confuse each other.

Subtraction: We will think of this as adding the additive inverse.

6-2=6+**-**2	-6-2=-6+**-**2	-6 - (-2)=-6+**-**(-2)
=4	=-8	=-6 + 2
		= -4

Manually change **-** to **+ -** until you think of subtraction as adding the additive inverse, or "adding the negative" as some say. (Realize, however, that expressions like -x are not necessarily negative.)

The "-" sign: This sign is used for several ideas.

1) It means "negative". Thus -2 means the number "negative two".
2) It means "the additive inverse of" or "the opposite of". Thus -(-3) means "the additive inverse of -3", which is 3.
3) When used between two expressions, it means subtraction, which we will think of as "adding the additive inverse". So 6-8 = 6+**-**8 = -2.

This helps us with expressions like 2 - (-(-9)). The innermost -9 mean "negative 9". Then (-(-9)) is the additive inverse of -9, which is 9. So we are left with 2 - 9, which we interpret as 2+**-**9 which is -7.

Note that the ideas in 1) and 2) can be equivalent. -2 (negative two) is fortunately the same thing as -2 (the additive inverse of two). Again realize, however, that expressions like -x are not necessarily negative. (If x is -7, then -x is positive.)

Note also that -3 and -1·3 are the same thing. It will be useful later to realize that the minus sign can always be replaced by -1· . Thus -x is -1·x or -1x, etc.

Division by zero:

Division by zero is undefined! Note that 6÷6=1 and 6÷1=6, but 6÷0 is not defined (because there is no number which multiplied by 0 gives 6). Alternatively, we could have defined division as multiplication by the multiplicative inverse (the reciprocal), and then dividing by 0 would be undefined because 0 has no multiplicative inverse.

Order of Operations and Grouping Symbols

( ), [ ], and the division bar are all **grouping symbols.**

Did you learn order of operations by memorizing "PEMDAS" (the acronym many students have used to remember "parentheses, exponents, multiplication, division, addition, subtraction")? If so, be careful! It should be interpreted as:

Parentheses (grouping symbols) innermost first. Within the grouping symbols:

1. Exponents
2. Multiplication and division, from left to right.
3. Addition [and subtraction], from left to right.

Example:	$-(-2-(9-2)) - 5^2 - 3$	Here's the original problem
	$-(-2-7) - 5^2 - 3$	We do the innermost grouping symbol (9-2) first
	$-(-9) - 5^2 - 3$	We do the remaining grouping (-2-7) symbol
	$-(-9) - 25 - 3$	Then we do the exponent 5^2
	9 + - 25 + -3	Simplifying and re-stating the subtraction as +-
	-16 + -3	Add
	-19	Then add

Note that the -5^2 involved squaring a **positive** 5, then making the resulting 25 negative (exponents first). If the problem had been intended to mean squaring a negative 5, the notation would have been $(-5)^2$.

Example:	$-(-22) \div 11$	Here's the original problem
	$-1 \cdot (-22) \div 11$	Interpret -x as -1·x. (So if x is -22, -x is -1· -22.)
	$22 \div 11$	Left to right, now that we have only multiplying and dividing
	2	

<u>Problems</u>

Simplify without using your calculator (or anyone else's).

1. $-12 \div 6 \cdot 2$
2. $3 \cdot (1 + 4^2)$
3. $-3 - 5 \cdot (-5)^2$
4. $-7 + -5 \div 3$
5. $-3 - 4^2 + -2$
6. $-5 - 25 \div 5$
7. $-9 \cdot (-5) - 3^2$
8. $(-3 - 3)^2 \div 4$
9. $-7 \cdot (-2) - 5$

10. Which axiom can be used to justify each of the following statements?

    A. 2 is a real number, and so is 5. Therefore, 7 is a real number.

    B. (2+5) ·3 is the same as 3·(2+5) *(Caution: did any grouping change?)*

    C. 0 + -1 is -1

    D. 1 + -1 is 0

    E. zero added to any number produces the same number

11. Classify $\frac{154}{\sqrt{121}}$ as fully as possible. (Use the following list: imaginary, real, rational, irrational, integer)

12. One number is six bigger than the other. Three times the larger one is 20 more than the smaller one. Find both numbers. *(Hint: call the smaller number x and the larger number x+6.)*

(Optional): Use the numbers 1, 2, 3, and 4 each once to replace the variables a, b, c, and d. What is the maximum possible value of the expression $a + b \cdot c^d$? (You may replace any letter with any one of the four values.)