Assignment #2: The Real Numbers

Numbers on the number line are the numbers we normally deal with; since they are used for counting and measuring real things, they are called the **real** numbers. Examples of <u>real</u> numbers are 2, -1.2, π, $\sqrt{2}$, $\frac{2}{7}$

Numbers can be thought of that are not in the set of real numbers, i.e. not on the number line. Some are called the **imaginary** numbers; these involve the square roots of negative numbers.

The real numbers can be classified as **positive** (greater than zero) or **negative** (less than zero). They can also be divided into the **rational** numbers and the **irrational** numbers. Rational numbers are those that can be expressed as a ratio of two whole numbers (positive or negative). In other words, a rational number can be expressed as a fraction, with whole numbers in the numerator and denominator. Irrational numbers cannot be expressed in this way. Examples of rational numbers are 2, -1.2, $\frac{2}{7}$ and examples of irrational numbers are $\sqrt{3}$, $-\sqrt[3]{5}$, π.

Question: "What about $1.\overline{7}$, with only the 7 repeating? Is that rational?"

Answer: Yes. $1.\overline{7}$ is the same as $\frac{16}{9}$. Try the division $16 \div 9$ to check this.

Question: "What about decimals that go on forever, but don't repeat? Are they rational?"
Answer: No. (How might you prove this?)

The rational numbers can be divided into the **integers** and the **non-integers**. The integers are the whole numbers, and their opposites. Note that zero is a whole number. Some examples of integers are 2, -3, and 0. The integers can be further described as **even**, or **odd**. The non-integers are the fractions between the integers.

Problems

1. What is the difference between an integer and a non-integer?
2. What is the difference between a real number and an imaginary number?
3. What is the difference between a rational number and an irrational number?
4. Show by example that by taking the square root of an integer, you could get any of the following kinds of numbers: imaginary, rational, irrational, integer. (That is, give an example of a square root that is imaginary, another square root that is rational, another square root that is irrational, etc.)
5. Mary found a number which is equal to 5 more than 13 times the number itself. Find this number and classify it into as many of the categories as you can. (Use the following list: imaginary, real, rational, irrational, integer, non-integer, positive, negative, even, odd.)
6. Prove that $5.\overline{9}$ is rational. (See the example below.)

Example: Prove that $1.\overline{7}$ is rational.

Let $x = 1.\overline{7}$. Then $10x = 17.\overline{7}$. So then 9x, which is 10x - x, is $17.\overline{7} - 1.\overline{7}$, which is 16. So 9x = 16, and thus $x = \frac{16}{9}$. So $1.\overline{7}$ is $\frac{16}{9}$, which is rational because it's the ratio of two integers.